Week 14 Worksheet - Vector

Instructions. Follow the instructions of your TA and do the following problems. You are not expected to finish all the problems. So take your time! :)

1. $\vec{a} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$. Which of the following expression are nonsense? Evaluate the sensible ones.

   (a) $3\vec{a} + \vec{b}$ $= \begin{pmatrix} 5 \\ 5 \\ 14 \end{pmatrix}$

   (b) $\vec{a} + \vec{c}$ ✗

   (c) $\vec{a} \cdot \vec{c}$ ✗

   (d) $\vec{a} - 2\vec{b}$ $\begin{pmatrix} 4 \\ -3 \\ 7 \end{pmatrix}$

   (e) $t\vec{a}$, where t is a real number. $\begin{pmatrix} 2t \\ t \\ 5t \end{pmatrix}$

   (f) $\vec{a}\vec{b}$ ✗

   (g) $\vec{a} + 5$ ✗

   (h) $2\vec{a} \cdot \vec{b} = -10$

2. Let $\vec{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Find s and t so that $\begin{pmatrix} 3 \\ 5 \end{pmatrix} = s\vec{a} + t\vec{b}$.

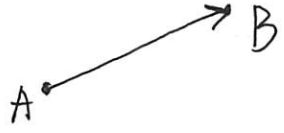
$$\begin{cases} 2s - t = 3 \\ 4s + t = 5 \end{cases}$$

$6s = 8 \Rightarrow s = \frac{8}{6} = \frac{4}{3}$

$t = 2s - 3 = \frac{8}{3} - 3 = -\frac{1}{3}$

3. Find a parametric equation for the line that passes through the points $A = (1, 0, 2)$ and $B = (3, 1, 4)$.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} t$$



4. Find the angle between $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

$$a \cdot b = |a||b| \cos\theta$$

$$6 = \sqrt{1^2+4+9}\sqrt{1^2+1^2+1^2}\cos\theta$$

$$\cos\theta = \frac{6}{\sqrt{14}\sqrt{3}} \qquad \theta = \arccos\frac{6}{\sqrt{14\cdot 3}} = \arccos\frac{6}{\sqrt{42}}$$

5. Let P be the plane containing the points $\overset{A}{(7,1,2)}$, $\overset{B}{(4,6,2)}$ and $\overset{C}{(3,3,3)}$. Find a parametrization of P.

$$\vec{r} = \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix} + s\begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix}$$

6. Does the plane containing the points $A = (1,0,0)$, $B = (0,1,0)$ and $C = (0,0,1)$ also contain the point $(1,1,1)$?

$$\vec{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

If $(1,1,1)$ is on the plane, then:

$$\begin{cases} 1 = 1 - t - s \\ 1 = t \\ 1 = s \end{cases}$$

No solution!

$\Downarrow$

$(1,1,1)$ is not on the plane.